Modeling of magnetic field perturbations on the balance-spring of a mechanical watch

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A magnetic field is a major enemy of a mechanical watch. This field may modify the structure of the balance-spring and change its resonance frequency. As a consequence, the watch loses its accuracy. The aim of this work is to quantify the impact of the magnetic field to the balance-spring resonance using a finite element approach. This coupled magneto-mechanical problem implies magnetic forces computation while accounting for a large rotation of the spring. An original algorithm including a dynamic mechanical time-stepping scheme is proposed to quantify the change of the resonance frequency.

***Index Terms*—Mechanical watch, resonance frequency, coupled magneto-mechanic problem.**

I. Introduction

A mechanical watch is the masterpiece of human creativity in mechanical engineering and arts. This watch uses only a mechanical system to measure the passage of time, as opposed to quartz watch that works electronically. A mechanical watch contains five main parts : main spring, gear wheel, escapement, balance-spring and display member [1]. Several parts are made of ferromagnetic materials. This work deals with the study of the impact of magnetic field on the balance-spring. Spring deformations due to magnetic perturbations -magnetostriction and magnetostatic stress- are evaluated by using a finite element approach. Moreover, we study the modification of both amplitude and frequency of first resonance resulting from this deformation.

II. Balance-spring of mechanical watch

The balance-spring is the heart of a mechanical watch and is the most sensitive part. It regulates the passage of time thanks to its oscillation and is responsible for the watch accuracy of the watch. The balance-spring considered in the simulation is shown in figure 1.

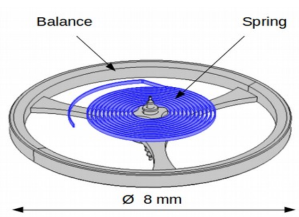

Fig. 1. Balance-Spring of a mechanical watch

The frequency of the oscillation depends on the geometry and the material of the balance-spring. A small modification in the structure of the spring changes the frequency of the resonance and generates an error on the time indication. The norm NIHS 91-10 requires that this error should not exceed 30 seconds per day [3]. Generally, the spring is made of iron-nickel alloys (Invar, Elinvar, Nivarox, …), notable for their low coefficient of thermal expansion [2]. These are ferromagnetic materials, and the structure can thus be impacted by magnetic fields: it has been experimentally shown that the presence of a magnetic field can be responsible of error of several minutes per day.

III. Magneto-mechanical problem

This study is placed in a weak-coupled magneto-mechanical problem. The magnetic problem on one side models the magnetic field in material, and the mechanical problem models the deformation resulting from the corresponding forces. To connect these two systems, we have to determine the equivalent forces related to magnetic origins. Figure 2 shows a general scheme of the considered magneto-mechanical modeling.

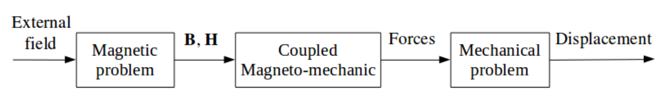

Fig. 2. Coupled magneto-mechanical problem

The magnetic equilibrium equation is represented by Maxwell's equations (neglecting the displacements currents) :

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \tag{1}$$

where $\mathbf{H}$ is the magnetic field, $\mathbf{B}$ the magnetic induction and $\mathbf{J}$ the current density. The magnetic problem is solved with a vector potential formulation. The mechanical equilibrium equation is given by Newton's second law :

$$\nabla \cdot \mathbb{T} + \mathbf{f} = \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} \tag{2}$$

where $\mathbb{T}$ is the stress tensor, $\mathbf{f}$ the driving force, $\mathbf{u}$ the displacement and ρ_m the mass density. The magnetic forces can be considered with two parts: magnetostatic forces and the magnetostriction effect [4]. Magnetostatic forces are related to the structure of the ferromagnetic material, and can be obtained by Maxwell's stress tensor [5]. The magnetostriction effect is a spontaneous strain in ferromagnetic material during the process of the magnetization [6]. The corresponding equivalent forces are deduced from the magnetostriction strain S_{kl}^{μ} that exhibits a non-linear behavior:

$$S_{kl}^{\mu} = \frac{\lambda_s}{M_s^2}\left(3M_k M_l - \delta_{kl}\|\mathbf{M}\|^2\right) \tag{3}$$

IV. Simulations

The first simulation considers the balance-spring placed in a uniform magnetic field and does not consider its oscillations. The aim is to determine the relative impact of magnetostriction and magnetostatic forces on its deformation. Figure 3 shows that the magnetostriction effect seems negligible for such application.

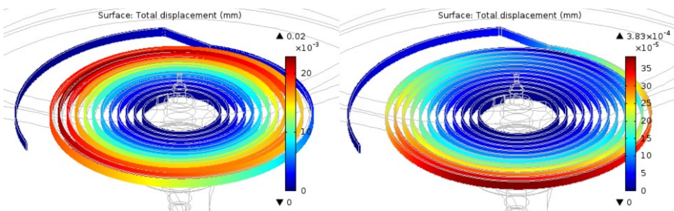

Fig. 3. Displacement of the balance-spring to due (a) magnetostatic forces and (b) magnetostriction effect.

The second study considers the oscillations of the balance-spring in the presence of the magnetic field. The original geometry of the balance-spring is simplified into a 2D geometry as shown in figure 4. The system is placed in a uniform magnetic field of 2400 A/m. As the balance-spring oscillations can not be considered as small displacements, the problem needs the consideration of geometric non-linearity (large rotation) associated to Arbitrary Lagrangian-Eulerian mesh (ALE).

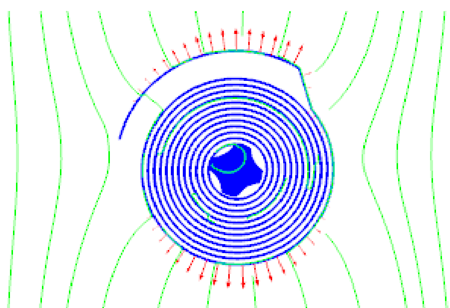

Fig. 4. Simulation system : (green) magnetic flux line (red) force density

The simulation starts with a static mechanical-only computation that gives an initial position to the balance-spring by imposing a chosen displacement. This allows to be in a pre-stress configuration. We take an initial position of 10°. The balance-spring is then released and oscillates around its equilibrium position. The second part of the simulation follows an iterative process including for each time step (0.1 ms) a magnetostatic computation, the expression of the corresponding magnetostatic forces, and the non-linear mechanical computation, with the mesh update. Figure 5 presents the simulation algorithm.

Figure 6 shows the oscillations of the balance-spring for H=0 A/m and 2400 A/m. We note that the magnetic field modifies both the amplitude and the frequency of the balance-spring oscillations. To analyze clearly the change of the frequency, a FFT with zero padding technique is considered. It shows that the frequency of resonance under the considered magnetic field is decreased by 0.02 Hz: the watch placed in such magnetic field will consequently lost 7.1 minutes per day.

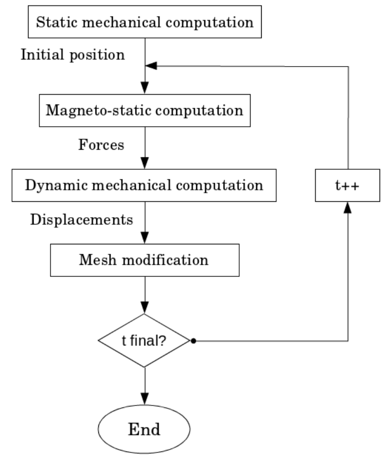

Fig. 5. Simulation algorithm

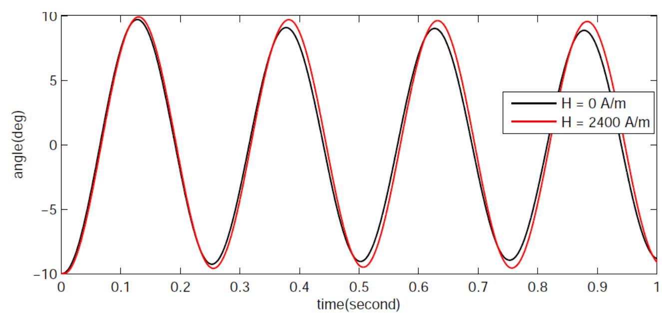

Fig. 6. Oscillations of the balance-spring with and without the presence of magnetic flied

V. Conclusion

The magnetic perturbation in the balance-spring is a serious problem for watchmakers. However, there is no accurate numerical modeling dealing with this problem. This contribution can provide informations about how a magnetic field perturbs the watch. It has been shown that the magnetic field may modify the resonance frequency of the balance-spring. Although for a short period this difference seems negligible, in one day it would give an inaccuracy of several minutes. For now, the dynamic work is limited to a 2D model. In future work, it is planed to study a more realistic 3D model.

VI. References

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